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Probability1 lessons and exercises

for first-year preparatory class students

Mme Amal ARGILLOS

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Contents

2. <u>Probable Space</u>

a. Definition of probability
b. Properties of probability
c. Probability on a set with equiproba
d. elementary events

3. <u>Conditional Probability</u>

a. Definition
b. Compound Probability Formula
c. Total Probability Formula
d. Bayes formula

4. <u>Stochastic Independence</u>

a. Independence of two events
b. Independence of several events

Chapter 1:

Combinatorial Analysis

The introduction

Combinatorial analysis provides counting (enumeration) methods that are particularly useful in probability theory.

Probability makes extensive use of set operations, so let us introduce at the outset the relevant notation and terminology.

A **set** is a collection of objects, which are the **elements** of the set. If S is a set and x is an element of S,. If x is not an element of S,. A set can have no elements, in which case it is called the **empty set**, denoted by O.

Sets can be specified in a variety of ways. If S contains a finite number of elements, say $x1, x2, \ldots, xn$, we write it as a list of the elements, in braces:

$S = \{x1, x2, \ldots, xn\}$.

For example, the set of possible outcomes of a die roll is $\{1, 2, 3, 4, 5, 6\}$, and the set of possible outcomes of a coin toss is $\{H, T\}$, where H stands for "heads" and T stands for "tails."

If S contains infinitely many elements $x1, x2, \ldots$, which can be enumerated in a list (so that there are as many elements as there are positive integers) we write

$S = \{x1, x2, \ldots\}$,

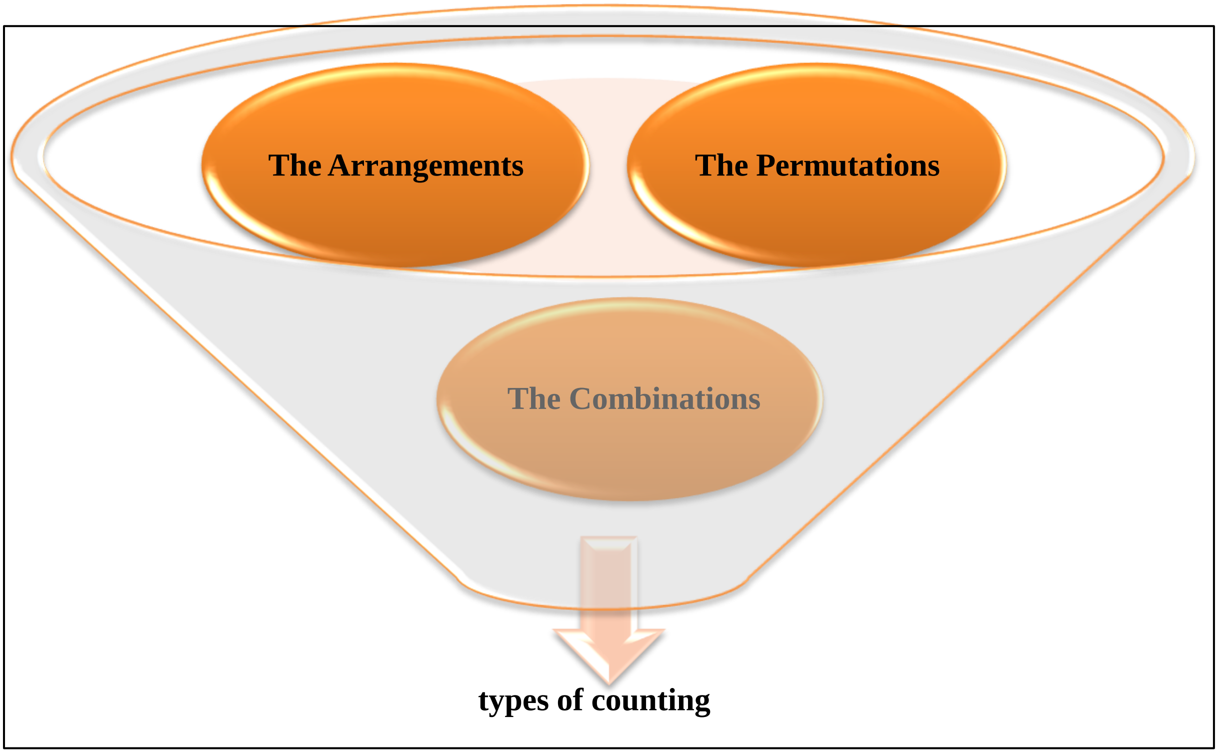
and we say that S is **countably infinite**. For example, the set of even integers can be written as $\{0, 2, -2, 4, -4, \ldots\}$, and is countably infinite.

Alternatively, we can consider the set of all x that have a certain property P, and denote it by $\{x \mid x \text{ satisfies } P\}$. (The symbol "|" is to be read as "such that.") For example the set of even integers can be written as $\{k \mid k/2 \text{ is integer}\}$. Similarly, the set of all scalars x in the interval [0, 1] can be written as $\{x \mid 0 \le x \le 1\}$. Note that the elements x of the latter set take a continuous range of values, and cannot be written down in a list (a proof is sketched in the theoretical problems); such a set is said to be **uncountable**.

If every element of a set S is also an element of a set T, we say that S is a **subset** of T, and we write $S \subset T$ or $T \supset S$. If $S \subset T$ and $T \subset S$, the two sets are **equal**, and we write $S = T$. It is also expedient to introduce a **universal set**, denoted by Ω, which contains all objects that could conceivably be of interest in a particular context. Having specified the context in terms of a universal set Ω, we only consider sets *S* that are subsets of Ω. (Tsitsiklis, 2000)

Combinatorial analysis provides counting (enumeration) methods that are particularly useful in probability theory.

There are three types of counting:



1. *Arrangements :*

Definition :

is the number of ways to choose a group of P elements in a set of n elements taking into consideration the order of these elements.

The arrangements of a set of elements correspond to the ordered sequences of certain elements of that set.

The arrangements of a set are characterized by the order of their constituent elements. For example, {(A,C)} and {(C,A)} are two different arrangements of the set {A,B,C}.(B,C)and(C,B)are two different arrangements of the set {B,C,D}In other words, the arrangements of the initial set of a random experiment correspond to the sampling space of all possible outcomes if this experiment has the following characteristics:

- Order matters in the experiment.
- The experiment involves either repetition or no repetition.
- The experiment involves certain elements from the initial set.

1-1 Arrangement without repetition

We call an arrangement without repetition an ordered sequence of P elements chosen from n and which cannot be repeated. Their number (which can be noted) is:

$$A_n^p = n!/(n-p)! \quad \text{withe } n \geq p$$

Example1

Let's calculate the number of Words withe 2-letters frome alle the liste of alphabet that do not contain the same letter twice.

Answer :

$$A_{26}^{2} = \frac{26!}{(26-2)!} = 25.26 = 650$$

Example 2

How many wards can we formed of 2 letters from this liste of {ABCDE} taking into consideration without repetition?

Answer :

$$A_{5}^{2} = \frac{5!}{(5-2)!}$$

1-2Arrangement with repetitions

An arrangement with repetitions of P objects taken from n is an ordered sequence of P elements chosen from n , and which can be repeated. This number is given by:

$$A_n^P = n^p \quad \text{with} \quad n \geq P$$

Example1

With the 26 letters of the alphabet, haw many words of 2 letters we can form with repetitions.

Answer :

$A^2_{26} = 26^2$

$= 676$ words

Example2

What is the theoretical capacity of an 8-digit telephone switchboard with repetitions?

Answer :

$A^8_{10} = 10^8$

$10^8 = 100000000$.

Example 3

Two marbles are drawn at random from a bag with replacement, taking into account their order. The bag contains a red marble (R), a blue marble (B), a yellow marble (Y), and a green marble (G). How many possible outcomes are there?

We can enumerate the elements of the sample space (Ω) and then count all the elements of this set. However, we notice that this random experiment has the following three characteristics:

(Ω),

We then count all the elements of this set. However, we notice that this random experiment has the following three characteristics:

- Order is important in this experiment.
- The experiment involves substitution.
- The experiment uses some elements from the initial set.

It is more efficient to calculate the number of possible outcomes using the formula for the number of arrangements with replacement of the initial set {R,B,Y,G}.Since this set contains 4 elements ($n = 4$), and we choose 2 from these ($p = 2$), we calculate the following , Since this set contains 4 elements ($n = 4$) , and we choose 2 of these ($p = 2$) , we calculate the following:

Number of arrangements

with ripitition $= n^p$

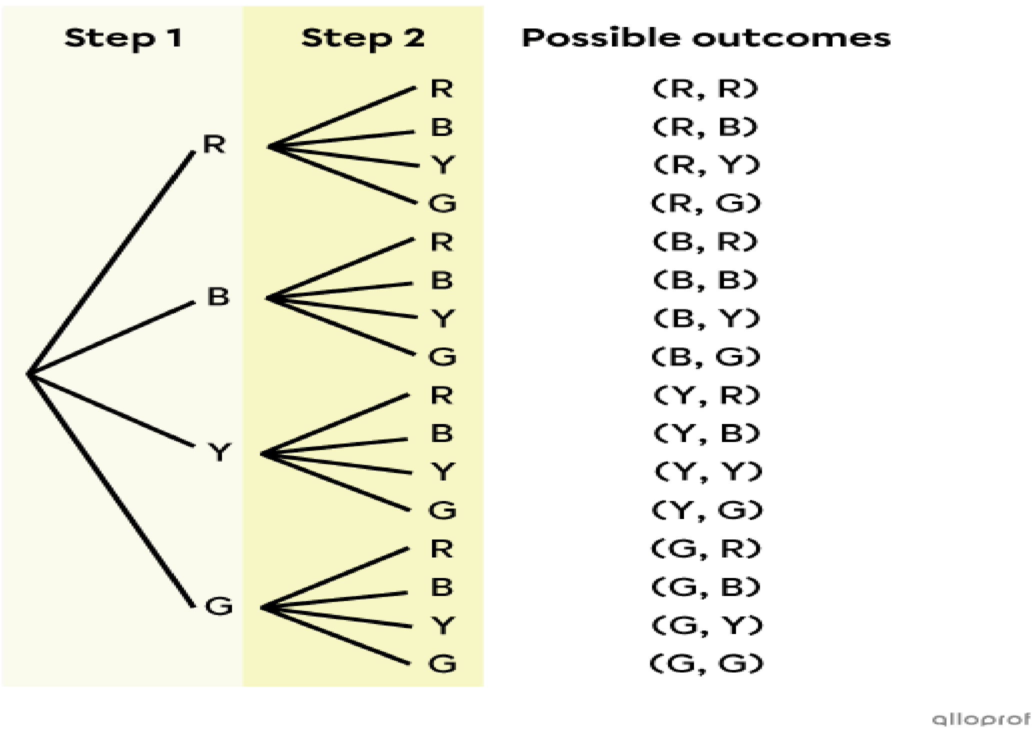
$= 4^2$

$= 16$

Answer: There are {16} possible outcomes in this random experiment.

16 Possible outcomes of this random experiment .

We can represent the sample space of the previous example using a tree diagram.



2-PERMUTATIONS

We call permutation of n elements any ordered set comprising these n elements.

The permutations of a set of elements correspond to the ordered sequences of all the elements of that set.

A distinction should be made between permutation without repetition and permutation with repetition.

The permutations of a set are characterized by the order of their constituent elements. For example, `(C, A, B)` and `(B, A, C)` are two different permutations of `{A, B, C}`.

In other words, the permutations of the initial set of a random experiment correspond to the sampling space of all possible outcomes if the experiment has the following characteristics.

- Order matters in the experiment.
- The experiment is irreplaceable.
- The experiment involves all elements of the initial set.

2-1 permutation without repetition

A permutation without repetition of n objects is one of the possible ways of ordering the n objects.

$$P^n{}_n = n!$$

Example 1

How many permutation can we classify

4 books in a shelf?

Answer :

4! = 4.3.2.1 = 24

So there are 24 ways to classify 4 books in a shelf .

Example 2

How many ways can we classify the 3 letters ABC?

Answer :

3! = 3.2.1 = 6 ways.

Example 3

All the marbles from a bag are drawn at random, without replacement, taking into account their order. The bag contains one red marble (R), one blue marble (B), one yellow marble (Y), and one green marble (G). How many possible outcomes are there?

We can enumerate the elements of the sample space Ω and then count all the elements of this set. However, we notice that this random experiment has the following three characteristics:

Ω,We then count all the elements of this set. However, we notice that this random experiment has the following three characteristics:

- Order is important in this experiment.
- The experiment is without replacement.
- The experiment involves all elements of the original set.

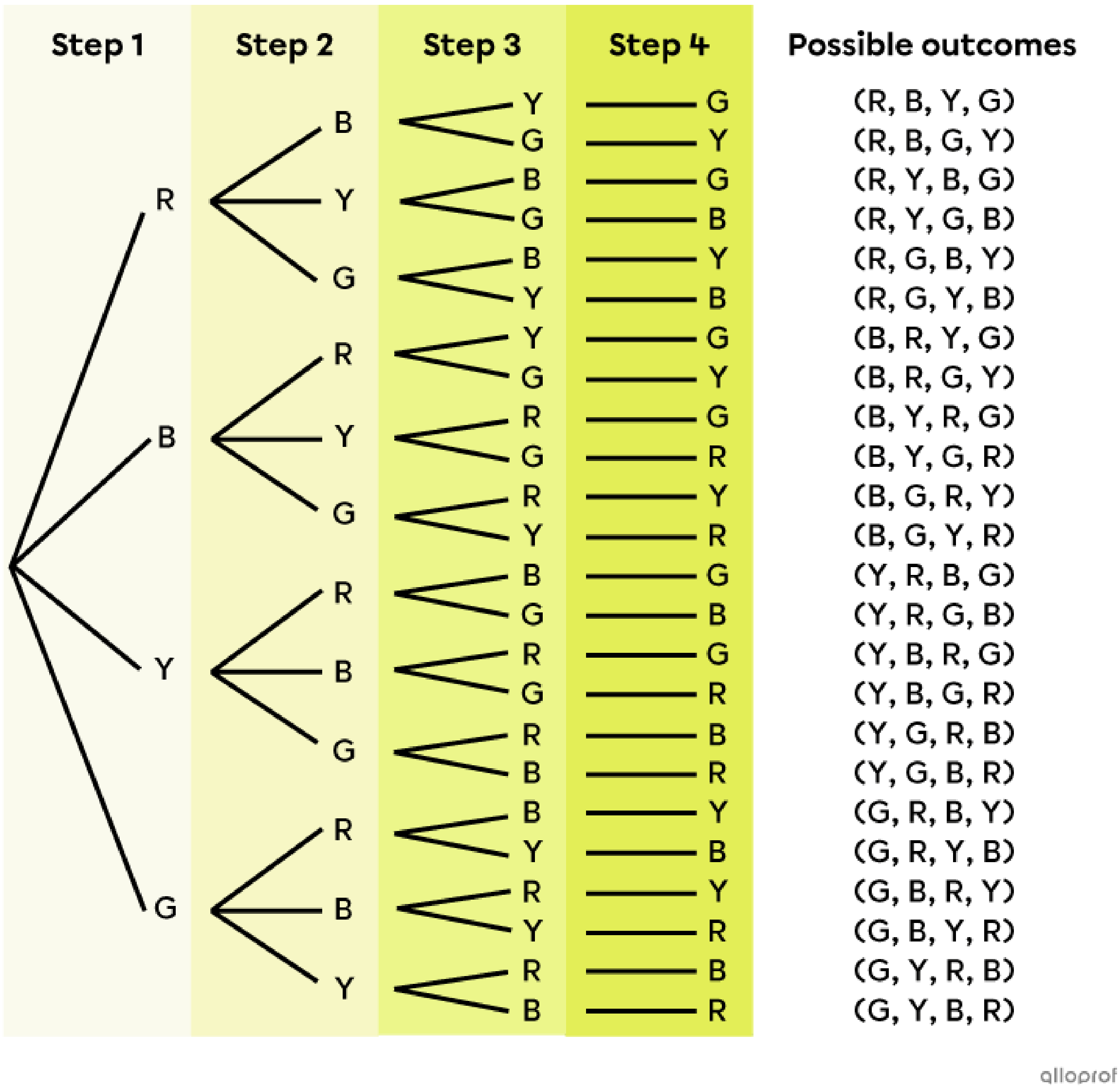
It is more efficient to calculate the number of possible outcomes using the formula for the number of permutations of the initial set {R,B,Y,G}. Since this set contains 4 elements (n=4), Since this set contains 4 elements
we calculate the following:

$P^4 = 4.3.2.1$

$= 24$

Answer: There are 24 possible outcomes in this random experiment.
24 Possible outcomes of this random experiment.

The sample space of the previous example can be represented using a tree diagram.



2-2 permutation with repetition

We consider n elements not all discernible (some are identical).

We divide these n elements into K discernible (different) groups. Elements of the same group are indistinguishable (identical). (DUSART, 2013)

We note n1, n2,n3, , , , nk the numbers of the K groups with

$$n_1 + n_2 + n_3 + \ldots\ldots + n_k = n$$

We call permutation with repetitions of these n elements an ordered arrangement of these n elements, their order is given by:

$$\boxed{P^n_{n_1, n_2, n_3, \ldots, n_k} = \frac{n!}{n_1!\, n_2!\, n_3! \ldots n_k!}}$$

Example1

How many ways can we arrange 5 white balls, 3 black balls and 2 red balls if we assume that the balls of the same color are indistinguishable?

Answer :

$$P_{(2,3,5)}^{10} = \frac{10!}{2!\,3!\,5!} = 2520$$

Example2

How many words (ordered) can we form with the letters of the word "Mississippi"?

Answer :

$$P_{(1,2,4,4)}^{11} = \frac{11!}{1!\,2!\,4!\,4!} = 34650$$

3- COMBINATIONS

Combinations of a set of elements correspond to ordered sequences of certain elements of that set.

The combinations of a set are not characterized by the order of their constituent elements. For example, (A,C) and (C,A) are two equivalent combinations of the set {A,B,C}. (DREYFUSS Pierre, 2015)

(UN,C) and (C,UN) are two equivalent combinations of the set {UN,B,C}

In other words, the combinations of the initial set of a random experiment correspond to the sampling space of all possible outcomes if this experiment has the following characteristics:

- The order is not important in the experiment.
- The experiment involves either replacement or no replacement.
- The experiment involves some elements from the initial set.

Example 1

Two marbles are drawn at random from a bag, without replacement and without regard to their order. The bag contains.

(R) one red marble

(B) one blue marble

(Y) one yellow marble

(G) and one green marble

How many possible outcomes are there?

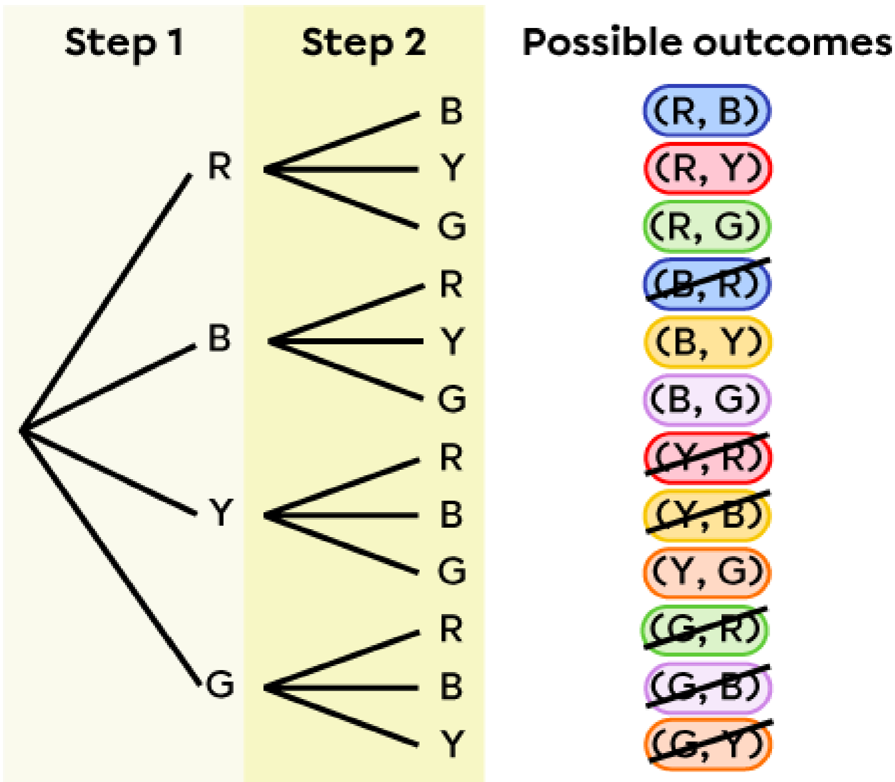
We could list the universe of possibilities (Ω)

Eliminate equivalent outcomes, then count the remaining elements in this set. Note, however, that this random experiment has the following three characteristics.

- The order is irrelevant in this experiment.
- The experiment is without replacement.
- The experiment involves some elements from the original set.

- From the beginning of the series
- {R, B, Y, G}, which contains 4 elements (n = 4), we choose 2 of them (k = 2). We calculate the following: (alloprof., 2011)

$C_4^2 = 6$



alloprof

3-1 combinations without repetition

it is the number of ways to choose a group of P elements from a set of n elements without taking into consideration the order of these elements. This number is given by:

$$C_n^P = n!/(n-p)!\,p! \quad \text{with } n \geq p$$

Example1

How many ways can you choose 2 letters from the 5{ A,B,C,D,E} without repetition ?

Answer :

$$C_5^2 = \frac{5!}{(5-2)!\,2!} = 10$$

Example2

How many ways can you choose 3 people out of 10?

Answer :

$$C_{10}^{3} = \frac{10!}{(10-3)!3!} = 120$$

Example 3

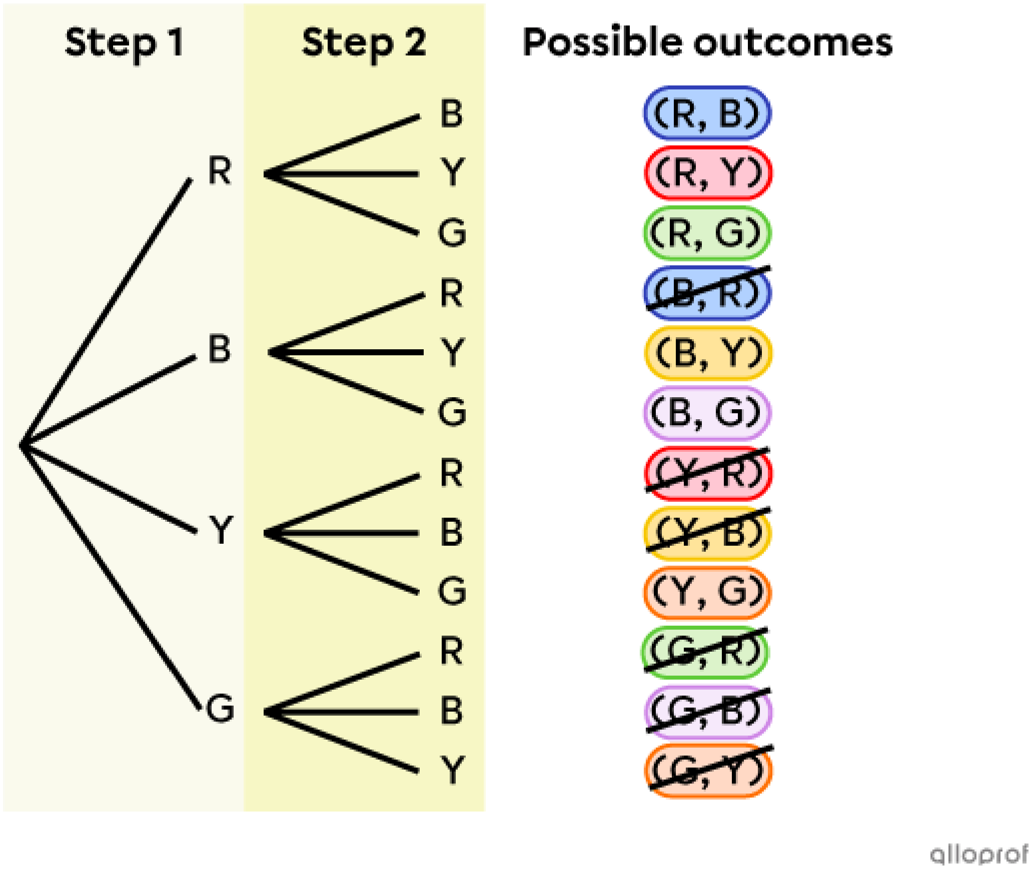
Two marbles are drawn at random from a bag, without replacement and without regard to their order. The bag contains one red marble, one blue marble, one yellow marble, and one green marble. How many possible outcomes are there? (alloprof., 2011)

We could list all the possibilities, eliminate equivalent outcomes, and then count the remaining elements. However, note that this random experiment has the following three characteristics:

- Order is irrelevant in this experiment.
- The experiment is without repetition .
- The experiment involves some elements from the original set.

$C_4^2 = 6$

We can represent the sample space of the previous example using a tree diagram.



Example 4

At his friend Karine's birthday party, Steve asks all his friends to say a few words of appreciation on camera to the birthday girl. Excluding Steve and Karine, there are 12 people at the party.

Considering that Steve wants each person to speak only once, find the number of different videos, according to the order in which friends appear, that Steve could film.

Answer :

note that this random experiment has the following three characteristics:

- Order is irrelevant in this experiment.
- The experiment is without repetition .
- The experiment involves some elements from the original set.

n=10 , P=1

$C_{10}^{1} = 10$

3-2 combinations with repetition

We call combination with repetition of " P " elements taken from n discernible elements, an unordered arrangement of these " P " elements with possible repetitions of one or more elements. This number is given by:

$$C_{n+p-1}^{P} = {(n+p-1)!}\big/{(n-1)!\,p!} \quad \text{with} \quad n \geq P$$

Example1

How many ways can you choose 2 letters from the 3 ABC with repetition?

Answer :

$$\boxed{C_{3+2-1}^{2} = \frac{4!}{(3-1)!\,2!} = 6}$$

conventions

- $C_n^1 = n$
- $c_n^p = c_n^{n-p}$
- $c_n^0 = c_n^n = 1$

4. Pascal's triangle and Newton's binomial

The following table is called Pascal's triangle. At the intersection of the line "P" and the column " n " we read the natural

$$\boxed{C_n^p}$$

To fill this triangle, we start by filling the column "P=0" with the numbers 1 ($c_n^0 = 1$ car) and the diagonal with the numbers 1 ($C_n^n = 1$ car)

We then complete using the relation:

$$C_n^p = C_{n-1}^{p-1} + C_{n-1}^p$$

So that is the sum of the number that is just above (C_{n-1}^P) and the number above and to the left (C_{n-1}^{P-1})

Pascal's Triangle

p not	0	1	2	3	4	5	6
1	**1**	**1**					
2	**1**	**2**	**1**				
3	**1**	**3**	**3**	**1**			
4	**1**	**4**	**6**	**4**	**1**		
5	**1**	**5**	**10**	**10**	**5**	**1**	
6	**1**	**6**	**12**	**20**	**15**	**6**	**1**

4-2- Newton's binomial:

The generalized binomial formula of classical remarkable identities

- For n= 2:

$$\boxed{(a+b)^2 = a^2 + 2ab + b^2}$$

- Which can be written as follows (see Pascal's triangle)

$$\boxed{(a+b)^2 = C_2^0 a^2 + C_2^1 ab + C_2^2 b^2}$$

- For n=3:

$$\boxed{(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3}$$

Or

$$\boxed{(a+b)^3 = C_3^0 a^3 + C_3^1 a^2 b + C_3^2 ab^2 + C_3^3 b^3}$$

And more generally

$$\boxed{(a+b)^n = C_n^0 a^n + C_n^1 a^{n-1} b + C_n^2 a^{n-2} b^2 + \cdots + C_n^n b^n}$$

<u>Or again :</u>

$$\boxed{(a+b)^n = \sum_{k=0}^{n} C_n^k a^{n-k} b^k}$$

Chapter 2:

Calculation of Probabilities

1 Probabilizable space

- A random experiment is an experiment whose outcome depends entirely on chance and whose possible outcomes are known.
- The sampling space (Ω) is the set of all possible outcomes of a random experiment.
- An event is a subset of the sampling space of a random experiment.
- A probability is a value between 0 and 1.

1 -1 RANDOM EXPERIENCES, EVENTS

- We call random experiment or test, any experiment whose result is governed by chance, when the experiment is repeated under the same conditions .

An event can involve a single outcome, several outcomes or all the outcomes in the sample space. It may also correspond to no outcome.

The probability of an event is the ratio of the number of favourable outcomes to the total number of possible outcomes of the random experiment.

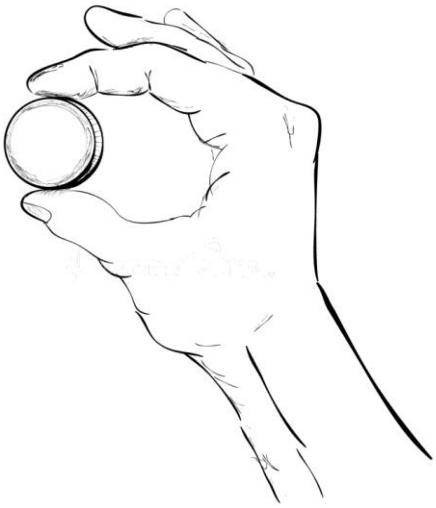
A random experiment can be performed in one step (simple random experiment) or in multiple steps (compound random experiment).

It is useful to employ different modes of representation (tree diagrams, Venn diagrams, etc.) and the multiplication rule to **enumerate** all the outcomes of a random experiment.

If a random experiment has several steps, the sample space is written by placing the outcomes of each step in brackets.

Example 1

The toss of a coin and the observation of the upper face and a random experience in the conditions.

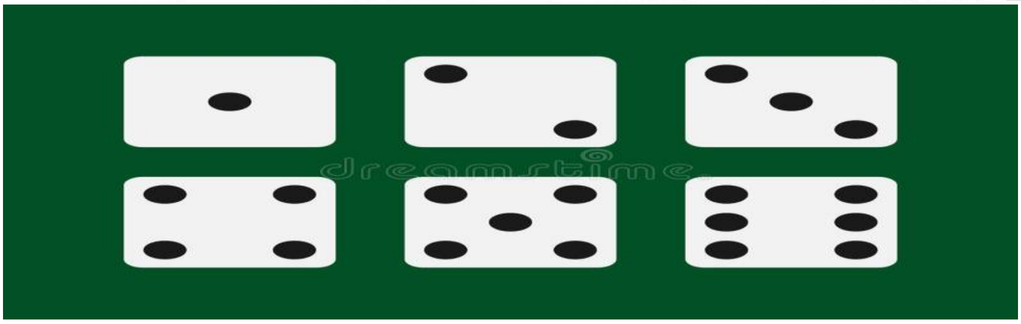


the fundamental set is given by :

Ω = { P,F}= 2cas

Example 2

Rolling a well-balanced, six-sided die and observing the top side is a random experiment that leads to "6" possible outcomes: 1, 2, 3, 4, 5, 6.



- we call fundamental set or universe or sample space or space of tests or even space of events, the set of all the results of a random experiment.

  We denote this space by Ω or S .

the fundamental set is given by:

$\Omega = \{ 1,2,3,4,5,6\} = 6$cas

Example 3

Two coins are tossed and we are interested in the possible outcomes.

This is a **2-step random experiment**.

The **sample space** is as follows:

$$\Omega=\{(T,T),(T,H),(H,T),(H,H)\}$$

where

T: Tails

H: Heads

Event B "Getting Heads at least once" is a subset of Ω. $B=\{(T,H),(H,T),(H,H)\}$

We **count** 3 favourable results for event B and 4 possible outcomes in the set Ω.

So the **probability** of getting Heads at least once is as follows: P(B)=Number of Favourable Outcomes Number of Possible Outcomes = 34

<u>An event A:</u>

is a set of results, or in other words, a subset of the fundamental set Ω

Is called a compound event or a simple event

- <u>The elementary event</u> : is an event which involves only one result of the experience.

- The event Ω which always occurs is called a certain event. Its probability is equal to 1.

- **<u>2-2 RELATIONS AND OPERATIONS ON EVENTS</u>**

- One can also combine events to form new events, using the various operations on sets:

- A U B: Union is the event that occurs if A or B is realized (or if A and B are both realized)

- A∩B : Intersection is the event that occurs if both A and B are realized.

- Cardinal of Ω : can be finite, cardinal of Ω is the element number of Ω .

- The event which never happens is called an impossible event and is denoted by ø

- A =B: Equality: events A and B are identical
- Ø: Event not possible
- Ω , S: Certain event
- Ā : Complementarity or contrary event of A in Ω which is realized when A is not realized and vice versa.
- A ∩ B = Ø : Elements A and B are incompatible
- **Example 1**

A roll of a 6-sided die constitutes a random experiment, A is the event of having an even number, B is the event of having an odd number, C is the element of having a multiple of "3".

- Calculate: A ∩ C , A ∩ B , B υ C , $\bar{C}$, A-C.

Answer :

A= {2 ,4,6}

B = {1,3,5}

C= {3,6 }

- A ∩ C = {6}= 1 case
- A ∩ B = { }
- B υ C = {1,3,5,6} =4 cases
- $\bar{C}$ = {1,2,4,5} = 4 cases

- A – C = {2,4} = 2cases

Example 2

A well-balanced coin is tossed three times.

Let A be the event corresponding to the consecutive appearance of two or more faces.

And B: the event corresponding to the consecutive appearance of three heads or three tails

- Calculate : A ∩ B , B U A , Ā , A - B

Answer :

- Ω = {PPP, PPF, PFP, PFF,FFF, FFP, FPF, FPP}=8 cas
- A= {PFF,FFF, FFP}= 3 cas
- B={PPP,FFF}=2 cases
- A Π B={FFF}=1 cas
- B U A = {PFF,FFF,FFP,PPP}=4 cas
- Ā=1 – A= {PPP, PPF, PFP, FPF, FPP}=5 cas
- A - B = {, PFF, FFP}=2 cas

3-2 Probability on a set with equiprobable elementary events

Quite often, the characteristics of an experiment suggest that equal probabilities are assigned to different outcomes in the fundamental set. Such a finite probability set Ω , where each element has the same probability, is called an equiprobable or uniform space. In particular, if Ω contains N points, the probability of each point is 1/ N.

On the other hand, if an event A is formed of n points, its probability is then n.1/ N= n/N. (Bernard, 2013)

Probability of an event = (number of ways it can happen) /(total number of outcomes)

P(A) = (Card A) / (Card Universe Ω)

$0 \leq n / N \leq 1$

Example1

What is the probability of tails when tossing a well-balanced coin?

a- Trial or random experiment: "Throw a balanced coin"

b- Event (A): "Get tails"

c- Probability calculation

P(A)=1/2=0.5

Example2

An urn contains a white ball (1W) and a black ball (1B).

We make 2 draws with replacement. What is the probability of having 2 quarter notes?

On a Ω = {(B, B) ;(W, W); (W, B); (B, W)} = 4 cases

Event A: "having 2 black balls": {(B, B} = 1 case

P (A) = n/N= 1/4

4-2 Consequences of the definition

- **$P(\bar{A}) = 1 - P(A)$**
- **$P(A - B) = P(A) - P(A \cap B)$**
- **$P(A \cup B) = P(A) + P(B) - P(A \cap B)$**
- **If $(A \cap B) = \emptyset \rightarrow P(A \cup B) = P(A) + P(B)$**

Example 1

Let A and B be events such that P(A) =3/8, P(B)=1/2 and P(A ∩ B)=1/4.

- Calculate : P(A U B), P($\bar{A}$), P(B), P($\bar{A}$ ∩ B) , P($\bar{A}$ U B), P(A ∩ B), P(B ∩ $\bar{A}$).
- **Solution:**
- P(A U B) = P(A) + P(B) – P(A ∩ B) = 3/8+1/2-1/4=5/8
- P($\bar{A}$)= 1- P(A)= 1-3/8=5/8
- P(B)=1 – P(B)= 1 – 1⁄2=1/2
- P(A ∩ B)= P(A U B)= 1- P(A U B)= 1- 5/8=3/8
- P(A U B)= P(A ∩ B) =1 - P(A ∩ B) = 1 – ¼= ¾
- P(A ∩ B)= P(A) - P(A ∩ B) = 3/8 – ¼ =1/8

- $P(B \cap \bar{A}) = P(B) - P(A \cap B) = ½ - ¼ = 1/4$
- **Example 3**

Two well-balanced dice are rolled, consider the following events:

A: {the sum of the numbers obtained on the 2 dice is equal to 3}

B: {the sum of the numbers obtained on the 2 dice is equal to 5}

C: {at least one of the two digits obtained is equal to 1}

Solution:

S = 36 cups;

A{(2,1),(1,2)}=2 Cas

B={(1,4), (4,1), (2,3), (3,2)}=4cas

C {(6,1),(5,1),(4,1),(3,1),(2,1),(1,6),(1,5),(1,4), (1,3),(1,2),(1,1)}=11 cas

(A∩C)= {(1,2), (2,1)}=2cas

(A∩B)={ }

(B∩C)= {(1,4), (4,1)}=2cas

P(A)=2/36,

P(B)=4/36,

P(C)=11/36.

Example 4

M&M sweets are of varying colours and the different colours occur in different proportions. The table below gives the probability that a randomly chosen M&M has each colour, but the value for tan candies is missing .

Colour	Brown	Red	Yellow	Green	Orange	Tan
Probability	0.3	0.2	0.2	0.1	0.1	?

(a) What value must the missing probability be?

(b) You draw an M&M at random from a packet. What is the probability of each of the following events?

1. You get a brown one or a red one.
2. You don't get a yellow one.
3. You don't get either an orange one or a tan one.

Solution:

The probabilities must sum to 1.0 Therefore, the answer is $1-0.3-0.2-0.2-0.1-0.1 = 1-0.9 = .1$.

(b) Simply add and subtract the appropriate probabilities.

P(Bu R)= 0.3 + 0.2 = 0.5

P(Y) = 1 − P(yellow) = 1 − 0.2 = 0.8.

P(O U T) = 1 − P(orange) − P(tan) = 1 − 0.1 − 0.1 = 0.8

4)This must happen; the probability is 1.0

Example1

Consider the experiment of rolling a die. Let A be the event 'getting a prime number', B be the event 'getting an odd number'. Write the sets representing the events

(1) A or B

(2) A and B

(3) A but not B

(4) 'not A'.

Solution:

S = {1, 2, 3, 4, 5, 6},

A = {2, 3, 5} and B = {1, 3, 5}

(1) A or B = $A \cup B$ = {1, 2, 3, 5}

(2) A and B = $A \cap B$ = {3,5}

(3) A but not B = A – B = {2}

(4) not A = A′ = {1,4,6}

2-3 - Conditional probability

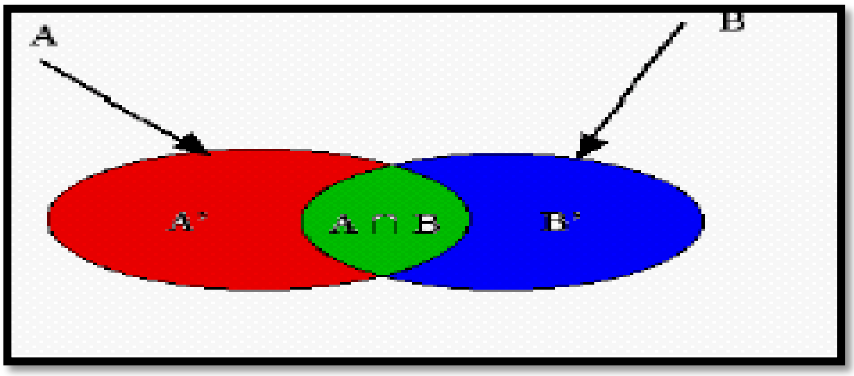
2-3 -1 Definition

Let A and B be two events of Ω , they can occur simultaneously: **A ∩ B ≠ ø**

A and B are two nonzero probabilities: **P(A)** > **0** and **P(B)** > **0.**

The probability of A knowing that B is realized is noted: **P(A/B)** and reads probability of A conditioned by B (or knowing B). (MUSSARD Stephane, 2014)

$$\mathbf{P(A/B)} = \frac{\mathbf{P(A \cap B)}}{P(B)}$$



Example 1

Consider the random experiment of rolling a six-sided die.

A is the "get a number greater than or equal to 4" event.

B is the "get even number" event.

Calculate P(A / B).

Example 2

In one local high school, 25% of students fail math, 15% fail chemistry, and 10% fail both math and chemistry.

We choose a student at random.

1- If the student failed in chemistry, what is the probability that he also failed in mathematics?

2 – If the student failed in mathematics, what is the probability that he also failed in chemistry?

3- What is the probability that he failed in mathematics or chemistry?

Solution:

1): $P(M/C)=\dfrac{P(M \cap C)}{P(C)} = 0.10/0.15 = 2/3 =$

2) : $P(C/M)=\dfrac{P(C \cap M)}{P(M)}=0.10/0.25=2/5$

3): $P(M \cup C)=P(M)+P(C) - P(M \cap C)$

$=0.25+0.15 – 0.10 =0.30$

Example 3

We know that 36% of homes have a dog and that in 22% of homes there is a dog and a cat. We also know that 30% of households have a cat.

1) What is the probability that a household has a dog or a cat?

2) What is the probability that a household owns a dog knowing that it owns a cat?

1) $P(D)= 0.36$

$P(D\cap C)= 0.22$

$P(C) =0.30$

1): $P(D \cup C) = P(D) + P(C) - P(D \cap C)$

$= 0.36 + 0.30 – 0.22$

$= 0.44$

2) : $P(D/C) = \frac{P(D \cap C)}{P(C)}$

$= 0.22/0.30$

$=0.73$

Example 4

using of a new "Kératine" cream for hair straightening. The results of the experiment revealed that **75%** of the male sample had a **positive result**, **10%** had an **allergy** and **5%** had a **positive result and an allergy**. If a man is randomly selected from the sample, what is the probability:

1- that he had a negative result?

2- that he had a positive result or an allergy?

3- that he had an allergy knowing that he had a positive result?

$P(p) = 0.75$

$P(A) = 0.10$

$P(p \cap A) = 0.05$

1) $P(N) = 1 - 0.75 = 0.25$

2) $P(+ \cup A) = P(+) + P(A) - P(+ \cap A)$

$= 0.75 + 0.1 - 0.05$

$= 0.80$

3) $P(A / p) = \dfrac{P(p \cap A)}{P(p)} = \dfrac{0.05}{0.75}$

$= 0.066$

Consequence of the definition:

By permuting A and B in the previous definition, it comes:

- $P(A \cap B) = P(A / B). P(B)$

- $P(B \cap A) = P(B / A) . P(A)$

2 Compound probability formula

For any events

$A_1, A_2, A_3, \ldots, A_n$

$P(A1 \cap A2 \cap A3 \cap \ldots \cap An) =$

$P(A1)P(A2/A1)P(A3/A1 \cap A2)\ldots\ldots P(An/A1 \cap A2 \cap A3 \cap \ldots \cap An\text{-}1)$

Example1

A lot contains 12 items which 4 are defective.

Three items are randomly drawn from the batch, one **after the other**.

Calculate the probability P that the three items are not defective.

Solution:

$$P(A) = \frac{A^1_8}{A^1_{12}} * \frac{A^1_7}{A^1_{11}} * \frac{A^1_6}{A^1_{10}}$$

$$P(A) = \frac{8}{12} * \frac{7}{11} * \frac{6}{10} = \frac{14}{55}$$

Example2

A class contains 15 boys and 10 girls. If we choose 04 students from the class at random, **one after the other**.

- Calculate the probability P to choose two girls then two boys.

Solution:

P(A) = $\frac{10}{25} * \frac{9}{24} * \frac{15}{23} * \frac{14}{22}$ = 0.062

2-3 - 3 Total probability formula

Suppose that the events

A 1 , A 2 , A 3 ,……………….A n form

a partition of a set

fundamental Ω ; the events

A i are therefore mutually exclusive

and their union is Ω . Let B be a

any other event. SO

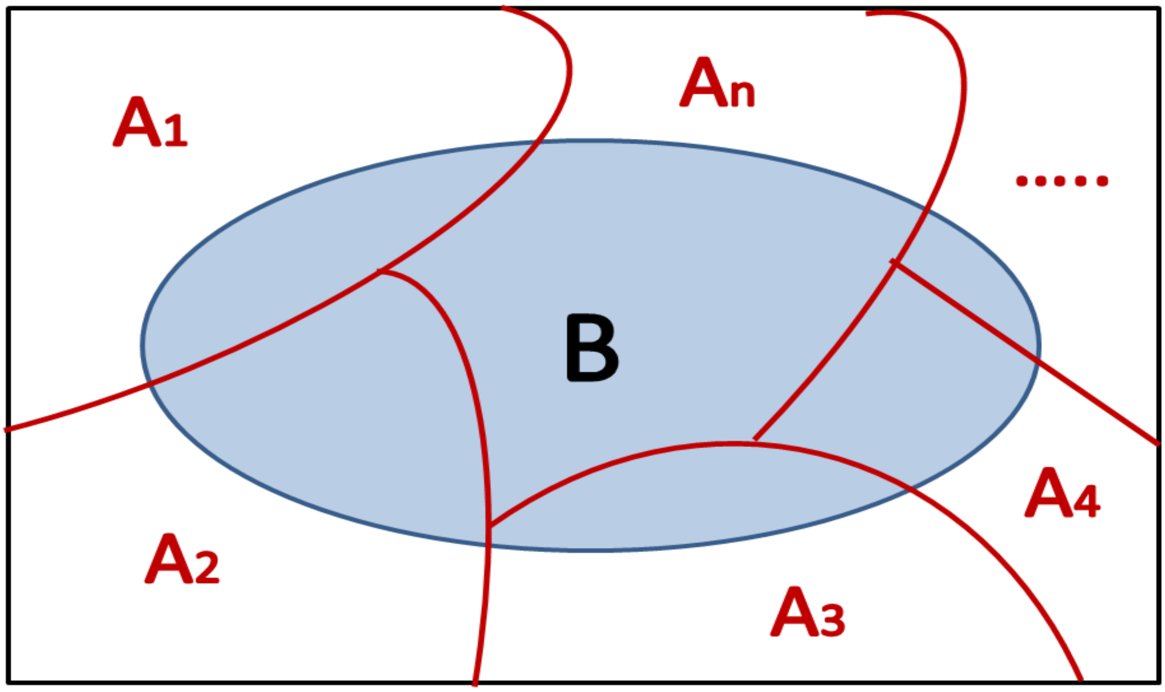
B= Ω ∩ B = **(A 1 AU 2 AU 3 ,… UA n)** ∩ B

B **= (A 1** ∩ **B) U (A 2** ∩ **B) U…. U(A n** ∩B **)**

Or the **A i** ∩ B are also mutually exclusive. Therefore,

$P(B)= P(A1 \cap B) + P(A2 \cap B) + \dots + P(An \cap B)$

▶ **$P(B)= P(B/A1)P(A1) + P(B/A2)P(A2) + \dots + P(B/An)P(An)$**



Example1

We have 4 urns U 1 , U 2, U 3, U 4 which respectively contain: 0 red balls and 5 white balls, 1 red and 4 white balls, 2 red and 3 white balls, 3 red and 2 white balls. We choose an urn at random and take a ball from it.

- What is the probability that it is red?

Solution:

$P(R)=P(R/U1)P(U1)+P(R/U2)P(U2)+P(R/U3)P(U3)+ P(R/U4)P(U4)$

$P(R) = 0.\ ¼ + 1/5.\ ¼ + 2/5.\ ¼ + 3/5.\ ¼ = 3/10.$

Example2

Three chests denoted C **1** , C **2** , C **3** each have two drawers, and in each drawer there is a coin. Chest C **1** contains 2 gold coins, C **2** 2 silver coins and C **3** one gold coin and one silver coin.

1. We open one of the C drawers at random, what is the probability that we will have found a silver coin?

Solution:

P(S)=P(S/C1)P(C1)+P(S/C2)P(C2)+P(S/C3)P(C3)

P(S)= 0.1/3+1/3.2/2+1/3.1/2

Example 3

A disease affects 3% of a given population. A screening test gives the following results:

- In sick individuals, 95% of tests are positive and 5% negative.
- In non-sick individuals, 1% of tests are positive and 99% negative.
- An individual is chosen at random.

1. Construct the tree of this random experiment.
2. What is the probability has. that he is sick and has a positive test?
3. that he is not sick and that he has a negative test?
4. that he has a positive test?
5. that he has a negative test?

6. Calculate the probability that he is not sick, knowing that the test is positive?
7. that he is sick, knowing that the test is negative?

Solution

This is a problem involving conditional probability. Let's define the events first:

- S: The individual is sick.
- N: The individual is not sick.
- P: The test result is positive.
- P': The test result is negative.

From the problem statement, we have the following initial probabilities:

- Probability of being sick: P(S) = 3\% = 0.03.
- Probability of not being sick: P(N) = 1 - P(S) = 1 - 0.03 = 0.97.

And the conditional probabilities:

- P(P|S) = 95\% = 0.95 (True Positive Rate or Sensitivity)
- P(P'|S) = 5\% = 0.05 (False Negative Rate)
- P(P|N) = 1\% = 0.01 (False Positive Rate)
- P(P'/N) = 99\% = 0.99 (True Negative Rate or Specificity)

2. What is the probability that he is sick and has a positive test?

- This is the probability of the intersection of events S and P, which is calculated using the multiplication rule for conditional probability:
  $P(S \cap P) = P(S) X P(P/S)$

Let's use a calculator to get an exact calculation.
The probability that the individual is sick and has a positive test is:

$P(S \cap P) = 0.03 X 0.95 = 0.0285$

3. What is the probability that he is not sick and that he has a negative test?

This is the probability of the intersection of events N and P', calculated as
$P(N \cap P') = P(N) X P(P/N)$
Let's use a calculator to get an exact calculation.
The probability that the individual is not sick and has a negative test is:
$P(N \cap P') = 0.97 X 0.99 = 0.9603$

4. What is the probability that he has a positive test?

The event of having a positive test (P) can happen in two mutually exclusive ways: being sick and testing positive ($S \cap P$), or not being sick and testing positive ($N \cap P$) We use the Law of Total Probability:
$P(P) = P(S \cap P) + P(N \cap P)$
We already found $P(S \cap P) = 0.0285$ Now we find $P(N \cap P) = P(N) X P(P/N)$
First, calculate $P(N \cap P)$
So, $P(N \cap P) = 0.97 X 0.01 = 0.0097$
Now, sum the probabilities:
The probability that the individual has a positive test is:
$P(P) = 0.0285 + 0.0097 = 0.0382$

5. What is the probability that he has a negative test?

The event of having a negative test (P') can happen in two mutually exclusive ways: being sick and testing negative

(S ∩P') or not being sick and testing negative (N∩P') We use the Law of Total Probability:

$P(P') = P(S \cap P') + P(N \cap P')$

We already found $P(N \cap P') = 0.9603$

$P(S \cap P') = P(S) \times P(P'/S)$

$P(S \cap P') = 0.03 \times 0.05 = 0.0015$

<u>Exemple 4</u>

A company has two factories A and B. Factory A produces 80% of the products and Factory B the remaining 20%. The proportion of products with defects is 0.05 for A and 0.01 for B.

1)What is the probability that a product chosen at random will have a defect?

<u>Solution:</u>

$P(D)= p\ (D/A)\ .\ p(A) + \ p\ (D/B).\ p\ (B) = 0,8.0.05+ 0,2.0,01 = \frac{3}{50}$

<u>Exemple 05</u>

The "Reading Challenge" competition is one of the most beautiful competitions taking place between Arab countries. Let's assume that 18 young participants competed in this competition, divide them into 10 boys and 8 girls. We choose at random 2 participants .

1-How many parts are there in total?

2-How many games are there mixed games?

2-We choose 4 participants at random, what is the probability of obtaining 2 boys and 2 girls?

<u>Solution</u>

E= {B=10 , G= 8 } , P=2

- $\mathbf{C^4_{18} =}$

$\mathbf{C^1_8 * C_{10}^1 =}$

$$\frac{C^2_8 * C^2_{10}}{C^4_{18}}$$

<u>Exemple 6</u>

Two well-balanced dice are rolled, consider the following events:

A = {the sum of the numbers obtained on the 2 dice is equal to 3}

B = {the sum of the numbers obtained on the 2 dice is equal to 5}

C ={at least one digit obtained out of the two is equal to 1}

1- Calculate: P (A / C) and P (A / B).

Solution

S= 36 cas ;

A2,1)،(1,2)}=)}2 Cas

B={(1,4), (4,1), (2,3), (3,2)}=4cas

C6,1)،(5,1)،(4,1)،(3,1)،(2,1)،(1,6)،(1,5)،(1,4)،(1,3)،(1,2)،(1,1)}=)}=11cas

(A∩ C)= {(1,2), (2,1)}=2cas

(A∩ B)= {}

(B∩ C)= {(1,4), (4,1)}=2cas

P(A)=2/36, P(B)=4/36, P(C)=11/36.

P (A / C) $= \frac{P(A\cap C)}{P(C)} = \frac{2/36}{11/36} = 2/11.$

According to the total probability theorem of Bayse

The theorem is visualized and proved, we are partitioning the sample space into a number of scenarios (events) *Ai*. Then, the probability that *B* occurs is a weighted average of its conditional probability under each scenario, where each scenario is weighted according to its (unconditional) probability.

One of the uses of the theorem is to compute the probability of various events *B* for which the conditional probabilities **P**(*B/Ai*) are known or easy to derive. (Pierre, ellipses)

The key is to choose appropriately the partition *A*1, . . . , *An*, and this choice is often suggested by the problem structure.

$P(B) = P(B/A_1)P(A_1) + P(B/A_2)P(A_2) + \ldots + P(B/A_n)P(A_n) \ldots (1)$

On the other hand, for a given i, the conditional probability of Ai, B being realized, is defined by

$$P(Ai/B) = \frac{P(Ai \cap B)}{P(B)}$$

In this equation, we replace p(B) by (1) and $P(Ai \cap B)$ by: $P(Ai \cap B) = P(Ai)$ $P(B/Ai)$, which gives

$$P(Ai/B) = \frac{P(B/Ai)\,P(Ai)}{P(B/A1)P(A1) + P(B/A2)P(A2) + \ldots + P(B/An)P(An)}$$

Here are some examples.

Example 1.

You enter a chess tournament where your probability of winning a game is 0.3 against half the players (call them type 1), 0.4 against a quarter of the players (call them type 2), and 0.5 against the remaining quarter of the players (call them type 3). You play a game against a randomly chosen opponent. What is the probability of winning?

Let *Ai* be the event of playing with an opponent of type *i*. We have

$\mathbf{P}(A1) = 0.5$, $\mathbf{P}(A2) = 0.25$, $\mathbf{P}(A3) = 0.25$.

Visualization and verification of the total probability theorem. The events $A1, \ldots, An$ form a partition of the sample space, so the event B can be decomposed into the disjoint union of its intersections $Ai \cap B$ with the sets Ai,

$B = (A1 \cap B) \cup \cdot \cdot \cdot \cup (An \cap B)$.

Using the additivity axiom, it follows that

$\mathbf{P}(B) = \mathbf{P}(A1 \cap B) + \cdot \cdot \cdot + \mathbf{P}(An \cap B)$.

Since, by the definition of conditional probability, we have

$\mathbf{P}(Ai \cap B) = \mathbf{P}(Ai)\mathbf{P}(B\,|Ai)$,

the preceding equality yields

$\mathbf{P}(B) = \mathbf{P}(A1)\mathbf{P}(B\,|A1) + \cdot \cdot \cdot + \mathbf{P}(An)\mathbf{P}(B\,|An)$.

For an alternative view, consider an equivalent sequential model, as shown on the right. The probability of the leaf $Ai \cap B$ is the product $\mathbf{P}(Ai)\mathbf{P}(B\,|Ai)$ of the probabilities along the path leading to that leaf. The event B consists of the three highlighted leaves and $\mathbf{P}(B)$ is obtained by adding their probabilities.

Let also B be the event of winning. We have

$\mathbf{P}(B\,|A1) = 0.3$, $\mathbf{P}(B\,|A2) = 0.4$, $\mathbf{P}(B\,|A3) = 0.5$.

Thus, by the total probability theorem, the probability of winning is

$\mathbf{P}(B) = \mathbf{P}(A1)\mathbf{P}(B\,|A1) + \mathbf{P}(A2)\mathbf{P}(B\,|A2) + \mathbf{P}(A3)\mathbf{P}(B\,|A3)$

$= 0.5 \cdot 0.3 + 0.25 \cdot 0.4 + 0.25 \cdot 0.5$

$= 0.375$.

Example 2

Alice is taking a probability class and at the end of each week she can be either up-to-date or she may have fallen behind. If she is up-to-date in a given week, the probability that she will be up-to-date (or behind) in the next week is 0.8 (or 0.2, respectively). If she is behind in a given week, the probability that she will be up-to-date (or behind) in the next week is 0.6 (or 0.4, respectively).

Alice is (by default) up-to-date when she starts the class. What is the probability that she is up-to-date after three weeks?

Let U_i and B_i be the events that Alice is up-to-date or behind, respectively, after i weeks. According to the total probability theorem, the desired probability $\mathbf{P}(U3)$ is given by

$$\mathbf{P}(U3) = \mathbf{P}(U2)\mathbf{P}(U3/U2) + \mathbf{P}(B2)\mathbf{P}(U3/B2) = \mathbf{P}(U2) \cdot 0.8 + \mathbf{P}(B2) \cdot 0.4.$$

The probabilities $\mathbf{P}(U2)$ and $\mathbf{P}(B2)$ can also be calculated using the total probability theorem:

$$\mathbf{P}(U2) = \mathbf{P}(U1)\mathbf{P}(U2/U1) + \mathbf{P}(B1)\mathbf{P}(U2/B1) = \mathbf{P}(U1) \cdot 0.8 + \mathbf{P}(B1) \cdot 0.4,$$

$$\mathbf{P}(B2) = \mathbf{P}(U1)\mathbf{P}(B2/U1) + \mathbf{P}(B1)\mathbf{P}(B2/B1) = \mathbf{P}(U1) \cdot 0.2 + \mathbf{P}(B1) \cdot 0.6.$$

Finally, since Alice starts her class up-to-date, we have

$$\mathbf{P}(U1) = 0.8, \mathbf{P}(B1) = 0.2.$$

We can now combine the preceding three equations to obtain

$$\mathbf{P}(U2) = 0.8 \cdot 0.8 + 0.2 \cdot 0.4 = 0.72,$$

$$\mathbf{P}(B2) = 0.8 \cdot 0.2 + 0.2 \cdot 0.6 = 0.28.$$

and by using the above probabilities in the formula for $\mathbf{P}(U3)$:

$$\mathbf{P}(U3) = 0.72 \cdot 0.8 + 0.28 \cdot 0.4 = 0.688.$$

Note that we could have calculated the desired probability $\mathbf{P}(U3)$ by constructing a tree description of the experiment, by calculating the probability of every element of $U3$ using the multiplication rule on the tree, and by adding. In experiments with a sequential character one may often choose between using the multiplication rule or the total probability theorem for calculation of various probabilities.

However, there are cases where the calculation based on the total probability theorem is more convenient. For example, suppose we are interested in the probability $\mathbf{P}(U20)$ that Alice is up-to-date after 20 weeks. Calculating this probability using the multiplication rule is very cumbersome, because the tree representing the experiment is 20-stages deep and has 220 leaves. On the other hand,with a computer, a sequential caclulation using the total probability formulas

$\mathbf{P}(Ui+1) = \mathbf{P}(Ui) \cdot 0.8 + \mathbf{P}(Bi) \cdot 0.4,$

$\mathbf{P}(Bi+1) = \mathbf{P}(Ui) \cdot 0.2 + \mathbf{P}(Bi) \cdot 0.6,$

and the initial conditions $\mathbf{P}(U1) = 0.8$, $\mathbf{P}(B1) = 0.2$

Example2:

Three machines A, B and C produce respectively 50%, 30% and 20% of the number of parts manufactured in a factory. The percentages of defective parts of these machines are 3%, 4%, and 5%. If we take a random part and it is defective, what is the probability that this part is produced by machine "A"?

Solution;

Let D be the event corresponding to a defective part.

$$P(A/D)= \frac{P(D/A)\,P(A)}{P(D/A)P(A) + P(D/B)P(B) + P(D/C)P(C)}$$

$$= \frac{(0.50)(0.03)}{(0.5)(0.03)+(0.3)(0.04)+(0.2)(0.05)}$$

$$= \frac{15}{37}$$

Exercise :

Three chests marked C1, C2, C3 each have two drawers, and in each drawer there is tow coins. The C1 vault contains 2 gold coins, C2 contains 2 silver coins and C3 one gold and one silver coin.

1. We open one of the 6 drawers at random, what is the probability that we have found a silver coin?

2. Randomly open one of the 6 drawers and find a silver coin. What is the probability that a drawer of chest C2 has been opened?

Solution:

1- P(S)=P(S/C1)P(C1)+P(S/C2)P(C2)+P(S/C3)P(C3)

P(R) = 0 + 1. 1/3 + ½.1/3 = 1/2.

2- P(C2/S)=

$$P(C2/S)= \frac{P(S/C2)P(C2)}{P(S/C1)P(C1)+P(S/C2)P(C2)+P(S/C3)P(C3)}$$

$$= \frac{1.\ 1/3}{0 + 1.1/3 + ½.1/3} = 2/3.$$

Exercise :

Three urns are given such that:

Urn A contains 6 red balls and 5 black balls.

Urn B contains 5 red balls and 1 black ball.

Urn C contains 5 red balls and 3 black balls.

We take an urn at random and we draw a ball from the urn.

1) If the ball drawn is red, what is the probability that it comes from urn "A"?

Solution;

$$P(A/R)= \frac{P(R/A)\,P(A)}{P(R/A)P(A)+P(R/B)P(B)+P(R/C)P(C)}$$

$$= \frac{(6/11)(1/3)}{(6/11)(1/3)+(5/6)(1/3)+(5/8)(1/3)}$$

$$= \frac{1584}{5819} = 0.2722$$

Exercise :

In a college in the Oran region, 4% of boys and 1% of girls are over 1.60m tall. We also know that 60% of students are girls.

If we take a student at random and if he is taller than 1.60m, what is the probability that this student is a girl?

This is a conditional probability problem. We will use Bayes' Theorem to find the required probability.

Let's define the events:

Solution

- G: The student is a Girl.
- B: The student is a Boy.
- T: The student is Taller than 1.60m

Given Probabilities

From the problem description, we have the following probabilities:

1. The percentage of students who are girls: P(G) = 60% = 0.60
2. The percentage of students who are boys (since students are either boys or girls) P(B) = 1 - P(G) = 1 - 0.60 = 0.40
3. The probability that a boy is taller than 1.60m

   P(T/B) = 4% = 0.04

4. The probability that a girl is taller than :1.60m

   P(T/G) = 1% = 0.01

Required Probability

We need to find the probability that the student is a girl, given that they are taller than 1.60m which is P(G/T)

P(G/T) = P(T|G) X P(G) / P(T)

Step 2: Calculate the Conditional Probability P(G|T)

Now we apply Bayes' Theorem using the values calculated:

P(G|T) = P(T ∩G)/P(T) = $\frac{0.006}{0.022}$

Let's use a calculator for the final division.

The probability that the student is a girl, given that they are taller than 1.60m , is: (G/T) = $\frac{0.006}{0.022}$ = $\frac{3}{11}$

As a decimal, this is approximately . 0.2727

The probability that the student is a girl, knowing that the test is positive (taller than 1.60m), is $\frac{3}{6}$ or about 27.27%.

Exercise :

Two urns U_1 and U_2 contain white balls and black balls.

- The U_1 urn contains 1 black ball and 9 white balls.

- The U_2 urn contains 4 black balls and 6 white balls.

An individual chooses one of the two urns, but as he is left-handed, he has 7 out of 10 chances of choosing the urn on the left, ie urn U_1.

Then in the chosen urn he draws a white ball, he shows it to you but you don't know which urn it comes from.

Determine the probability that this ball was extracted from the urn U_1.

Solution

This problem requires using Bayes' Theorem to find a conditional probability. We want to find the probability that the ball came from Urn U1, given that the drawn ball is white, which is written as P(U1/W)

Step 1: Define the Probabilities

Let be U1 the event of choosing Urn 1,

and be U2 the event of choosing Urn 2.

Let W be the event of drawing a white ball.

Prior Probabilities (Choosing the Urn):

The individual chooses U1 with a 7 out of 10 chance:

$P(U1) = \frac{7}{10} = 0.7$

The probability of choosing U2 is

P(U2) = 1 - P(U1) = 1 - 0.7 = 0.3

Conditional Probabilities (Drawing White given the Urn):

- Urn U1 has 1 black and 9 white balls (Total 10).

  $P(W /U1) = \frac{9}{10} = 0.9$

- Urn U2 has 4 black and 6 white balls (Total 10).

$P(W / U2) = \frac{6}{10} = 0.6$

Step 2: Calculate the Total Probability of Drawing a White Ball, **P(W)**

We use the Law of Total Probability:

$P(W)= P(W / U1)\, P(U1) + P(W / U2)\, P(U2)$

Let's use a calculator to get the exact values.

The total probability of drawing a white ball is P(W) = 0.81

Step 3: Apply Bayes' Theorem

We want to find P(U1 / W)

$$P(U1 /W) = \frac{P(W / U1)\, P(U1)}{P(W)}$$

$$= \frac{0.63}{0.81}$$

$$= \frac{7}{9}$$

$= 0.77$

Bayse formulat

Let $A1, A2, \ldots, An$ be disjoint events that form a partition of the sample space, and assume that **P**(Ai) > 0, for all i. Then, for any event B such that **P**(B) > 0, we have

$$\mathbf{P}(Ai/B) = \frac{\mathbf{P}(Ai)\mathbf{P}(B/Ai)}{\mathbf{P}(B)}$$

$$= \frac{\boldsymbol{P}(Ai)\boldsymbol{P}(B/Ai)}{\mathbf{P}(A1)\mathbf{P}(B/A1) + \cdot \cdot \cdot + \mathbf{P}(An)\mathbf{P}(B/An)}$$

.

To verify Bayes' rule, note that **P**(Ai)**P**(B/Ai) and **P**(Ai/B)**P**(B) are equal, because they are both equal to **P**($Ai \cap B$). This yields the first equality.
The second equality follows from the first by using the total probability theorem to rewrite **P**(B).
Bayes' rule is often used for **inference**. There are a number of "causes" that may result in a certain "effect." We observe the effect, and we wish to infer the cause. The events $A1, \ldots, An$ are associated with the causes and the event B represents the effect. The probability **P**(B/Ai) that the effect will be observed when the cause Ai is present amounts to a probabilistic model of the cause-effect relation Given that the effect B has been observed, we wish to evaluate the (conditional) probability **P**(Ai/B) that the cause Ai is present.

An example of the inference context that is implicit in Bayes' rule. We observe a shade in a person's X-ray (this is event B, the "effect") and we want to estimate the likelihood of three mutually exclusive and collectively exhaustive potential causes: cause 1 (event $A1$) is that there is a malignant tumor, cause 2 (event $A2$) is that there is a nonmalignant tumor, and cause 3 (event $A3$) corresponds to reasons other than a tumor. We assume that we know the probabilities $\mathbf{P}(Ai)$ and $\mathbf{P}(B/Ai)$, $i = 1, 2, 3$. Given that we see a shade (event B occurs), Bayes' rule gives the conditional probabilities of the various causes as

$$\mathbf{P}(Ai|B) = \frac{\mathbf{P}(Ai)\mathbf{P}(B/Ai)}{\mathbf{P}(A1)\mathbf{P}(B/A1) + \mathbf{P}(A2)\mathbf{P}(B/A2) + \mathbf{P}(A3)\mathbf{P}(B/A3)}$$

$i= 1, 2, 3.$

For an alternative view, consider an equivalent sequential model, as shown on the right. The probability $\mathbf{P}(A1/B)$ of a malignant tumor is the probability of the first highlighted leaf, which is $\mathbf{P}(A1 \cap B)$, divided by the total probability of the highlighted leaves, which is $\mathbf{P}(B)$.

Example 1

Three machines A, B and C produce respectively 50%, 30% and 20% of the number of parts manufactured in a factory. The percentages of defective parts of these machines are 3%, 4%, and 5%. If we take a random part and it is defective, what is the probability that this part is produced by machine "A"?

Solution;

Let D be the event corresponding to a defective part.

$$P(A/D)= \frac{P(D/A)\,P(A)}{P(D/A)P(A)+P(D/B)P(B)+P(D/C)P(C)}$$

$$= \frac{0.50x0.03}{(0.5x0.03)+(0.3x0.04)+(0.2x0.05)}$$

$$= \frac{15}{37}$$

Exercise :

Three chests marked C1, C2, C3 each have two drawers, and in each drawer there is tow coins. The C1 vault contains 2 gold coins, C2 2 silver coins and C3 one gold and one silver coin.

1. We open one of the 6 drawers at random, what is the probability that we have found a silver coin?

2. Randomly open one of the 6 drawers and find a silver coin. What is the probability that a drawer of chest C2 has been opened?

Solution:

1- P(S)=P(S/C1)P(C1)+P(S/C2)P(C2)+P(S/C3)P(C3)

P(R)= 0 + 1. 1/3 + ½.1/3 = 1/2.

2- P(C2/S)=

$$P(C2/S) = \frac{P(S/C2)P(C2)}{P(S/C1)P(C1)+P(S/C2)P(C2)+P(S/C3)P(C3)}$$

$$= \frac{1.\ 1/3}{0 + 1.1/3 + ½.1/3} = 2/3.$$

Exercise :

Three urns are given such that:

Urn A contains 6 red balls and 5 black balls.

Urn B contains 5 red balls and 1 black ball.

Urn C contains 5 red balls and 3 black balls.

We take an urn at random and we draw a ball from the urn.

1) If the ball drawn is red, what is the probability that it comes from urn "A"?

Solution;

$$P(A/R) = \frac{P(R/A)\ P(A)}{P\ (R/A)P(A) +P(R/B)P(B) +P(R/C\ )P(C)}$$

$$= \frac{(6/11)(1/3)}{(6\ /11)(1/3)+(5/6)(1/3)+(5/8)(1/3)}$$

= 0.2722

Chapter 4:

Stochastic Independence

1) Independence of two events:

We have introduced the conditional probability **P**(A/B) to capture the partial information that event B provides about event A. An interesting and important special case arises when the occurrence of B provides no information and does not alter the probability that A has occurred (BRAHAMI 2018) .,

P(A/B) = **P**(A).

When the above equality holds, we say that A is **independent** of B. Note that by the definition **P**(A|B) = **P**(A ∩ B)/**P**(B), this is equivalent to
P(A ∩ B) = **P**(A)**P**(B).
We adopt this latter relation as the definition of independence because it can be used even if **P**(*B*) = 0, in which case **P**(A/B) is undefined. The symmetry of this relation also implies that independence is a symmetric property; that is, if A is independent of B, then B is independent of A, and we can unambiguously say that A and B are **independent events**.
Independence is often easy to grasp intuitively. For example, if the occurrence of two events is governed by distinct and no interacting physical processes, such events will turn out to be independent. On the other hand, independence is not easily visualized in terms of the sample space. A common first thought is that two events are independent if they are disjoint, but in fact the opposite is true: two disjoint events A and B with **P**(A) > 0 and **P**(B) > 0 are never independent, since their intersection A ∩ B is empty and has probability 0.

Let A and B be two events in a probability space Ω such that p(A) > 0 and p(B) > 0.

We will say that A and B are independent or stochastically independent if and only if:

1- **P(A/B) = P(A)**

This means that knowing that B has happened does not influence the realization of event A.

$$2 - P(A \cap B) = P(A) * P(B)$$

Or

This relation is used very often as a definition of the independence of two events.

Example1

Assuming the equiprobability of the possible distributions into boys and girls of a family of 3 children, study the independence of the events:

A "the first child is a girl"

B "the third child is a boy"

<u>Solution :</u>

Ω = {BBB, BBG, BGB, BGG, GBB, GGB, GBG, GGG}=8cases

A = {GBB, GGB, GBG, GGG}=4 cases

B= {BBB, BGB, GBB, GGB,}=4 cases

$A \cap B = \{GBB, GGB\}$=2 cases

$P(A)= 4/8$, $P(B)= 4/8$, $P(A \cap B)= 2/8$

$P(A).\ P(B)= 4/8\ .\ 4/8 =16/64 =2/8 = P(A \cap B)$

So events A and B are independent

Example2

A perfectly balanced six-sided die is thrown. Justify the independence of the events A = "we obtain the draw 2, 4 or 6" and B = "we obtain the draw 3 or 6"

Solution :

$P(A) = 1/2$,

$P(B) = 1/3$

$P(A \cap B) = \{6\} = 1/6$

So

$P(A \cap B) = P(A) \times P(B)$

Events A and B are independent

Example3

We give the distribution of the 2,000 employees of a company.

An employee of this company is randomly interviewed. We notice :

- O the event: "The employee questioned is a worker";

- T the event: "The employee questioned is a technician";

- H the event: "The employee questioned is a man";

1. Are events O and H independent?

Are events T and H independent?

Solution

	Workers	**Technicians**	**Engineers**	**Total**
Men	**760**	**609**	**371**	**1,740**
Women	**40**	**91**	**129**	**260**
Total	**800**	**700**	**500**	**2 000**

Q1

P(O) = 800/2000,

P(H) = 1740/2000

and

P(O ∩ H) = 760/2000 = 0.38

P(O ∩ H) = P(T) × P(H) = 0.348

SO

Events O and H are not independent

Solution Q 2:

P(T) = 700/2000,

P(H) = 1740/2000 and

$P(T \cap H) = 609/2000 = 0.3045$

$P(T \cap H) = P(T) \times P(H) = 0.3045$

SO

The events T and H are independent

Example

Consider an experiment involving two successive rolls of a 4-sided die in which all 16 possible outcomes are equally likely and have probability 1/16.

(a) Are the events

Ai = {1st roll results in i},

Bj = {2nd roll results in j},

independent? We have

$\mathbf{P}(A \cap B) = \mathbf{P}$ (the result of the two rolls is (i, j)) $= \frac{1}{16}$

$$\mathbf{P}(A_i) = \frac{\text{number of elements of } A_i}{\text{total number of possible outcomes}}$$

$= \frac{4}{16}$

P(*Bj*) = <u>number of elements of *Bj*</u>
total number of possible outcomes

$= \frac{4}{16}$

We observe that **P**(Ai ∩Bj) = **P**(Ai)**P**(Bj), and the independence of Ai and Bj is verified. Thus, our choice of the discrete uniform probability law (which might have seemed arbitrary) models the independence of the two rolls.

(b) Are the events

A = {1st roll is a 1}, B= {sum of the two rolls is a 5},

independent? The answer here is not quite obvious. We have

P(*A* ∩ *B*) = **P (** the result of the two rolls is (1,4))

$= \frac{1}{16}$

and also

P(*A*) = <u>number of elements of *A*</u>
total number of possible outcomes

$= \frac{4}{16}$

. The event *B* consists of the outcomes (1,4), (2,3), (3,2), and (4,1), and

P(*B*) = <u>number of elements of *B*</u>
total number of possible outcomes

$= \frac{4}{16}$

Thus, we see that **P**(*A* ∩ *B*) = **P**(*A*)**P**(*B*), and the events *A* and *B* are independent.

(c) Are the events

A = {maximum of the two rolls is 2}, B= {minimum of the two rolls is 2},

independent? Intuitively, the answer is "no" because the minimum of the two rolls tells us something about the maximum. For example, if the minimum is 2, the maximum cannot be 1. More precisely, to verify that *A* and *B* are not independent, we calculate

P(*A* ∩ *B*) = **P** **(**the result of the two rolls is (2,2))

$$= \frac{1}{16}$$

and also

$$\mathbf{P}(A) = \frac{\text{number of elements of } A}{\text{total number of possible outcomes}}$$

$$= \frac{3}{16}$$

$$\mathbf{P}(B) = \frac{\text{number of elements of } B}{\text{total number of possible outcomes}}$$

$$= \frac{5}{16}$$

<u>Example</u>.

Throw 2 dice

- A first die lands 1
- B second die shows larger number than first die
- C both dice show same number

This problem involves calculating probabilities based on the outcomes of throwing two standard six-sided dice. The total number of possible outcomes is 6x6 = 36 We will denote the outcome of the first die as d1 and the outcome of the second die as d2 .

- Event A: The first die lands 1

Event A is the set of outcomes where d_1 = 1

A = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6) }

The number of outcomes in A = 6

The probability of event A is:

P(A) =$\frac{6}{36}$ = $\frac{1}{6}$

- Event B: The second die shows a larger number than the first die

We list the outcomes where the second die is strictly greater than the first:

- If d1 = 1 : { (1, 2), (1, 3), (1, 4), (1, 5), (1, 6) } (5 outcomes)
- If d1 =2 :{ (2, 3), (2, 4), (2, 5), (2, 6) } (4 outcomes)
- If d1 = 3 :{(3, 4), (3, 5), (3, 6)} (3 outcomes)
- If d1 = 4 :{(4, 5), (4, 6)} (2 outcomes)
- If d1 = 5 :{(5, 6)} (1 outcomes)
- If d1 = 6 :(0 outcomes)

  The total number of outcomes in B is 5+4+3+2+1 = 15

  P(B) = $\frac{15}{36}$

- Event C: Both dice show the same number (d1=d2)

Event C is the set of outcomes where the numbers are equal (doubles):

C = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) }

The number of outcomes in C = 6

The probability of event C is:

$P(C) = \frac{6}{36} = \frac{1}{6}$

Since the question asks to "Throw 2 dice" and then lists three conditions (A, B, C), it is asking for the probability of each individual event based on the sample space of throwing two dice.

If you need to calculate the probability of any combination of these events (like and , or or), please specify which combination you are interested in. For example, the probability that the first die is 1 **and** the second die is larger than the first ($A \cap B$)

($A \cap B$) = {(1, 2), (1, 3), (1, 4), (1, 5), (1, 6) }

$P(A \cap B) = \frac{5}{36}$

2) Independence of several events :

A sequence of events A_1 , A_2 , A_3 ,…A_i…………….A_n linked to a random experiment are mutually independent if:

$P(A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_n) = P(A_1)\, P(A_2)\, P(A_3) \ldots\ldots P(A_n)$

Exercices

Exercise 01

1) Calculate the number of words (three-letters) that can be formed from the following six letters { A,B,C,D,E,F } .
2) Same question but choosing distinct (different) letters.

Solution

This problem involves counting principles, specifically permutations and arrangements with and without repetition. We have a set of n=6 distinct letters: {A, B, C, D, E, F}

We are forming a word of length k=3

1) Calculate the number of three-letter words that can be formed (repetition allowed)

Since the problem does not specify that the letters must be distinct, we assume that repetition is allowed. For each of the three positions in the word, we have 6 choices.

- Choices for the first letter: 6
- Choices for the second letter: 6
- Choices for the third letter: 6

The total number of words is the product of the choices for each position.
Let's use a calculator to find the result.
The number of three-letter words that can be formed when repetition is allowed is 216 .

2) Same question but choosing distinct (different) letters

When the letters must be distinct, this is a permutation problem, denoted as $P(n, k)$, where $n=6$ and $k=3$

- Choices for the first letter: 6
- Choices for the second letter (must be different from the first): 5
- Choices for the third letter (must be different from the first two): 4

The total number of words is the product of the choices for each position:

$P(6, 3) = 6 \times 5 \times 4$

Let's use a calculator to find the result.

The number of three-letter words that can be formed when using distinct letters is 120.

Exercise 02

1) How many permutations of the 4 objects (a,b,c,d) are there ?

Solution

The number of permutations of n distinct objects is given by n!

For the 4 objects {a, b, c, d} we need to calculate 4!

Let's use a calculator to get the exact value.

There are 4! Permutations

$4! = 4 \times 3 \times 2 \times 1 = 24$

There are **24** permutations of the 4 objects

Exercise 03

We must put 2 different mathematics books (M), 4 different economics books (E) and 6 different accounting books (C) on a shelf.

1) How many different storage options do we have ?
2) If the books must be arranged by specialty how many different storage possibilities are there?

Solution

Step 1: Understanding and Analyzing the Problem

1) For the first question, we are looking for the total number of ways to arrange all the books without constraints. We have 2 math books, 4 economics books, and 6 accounting books, for a total of:
2 + 4 + 6 = 12 books.

2) For the second question, the books must be grouped by subject. We must therefore consider the possible arrangements within each group (math, economics, accounting) and also the order of the groups themselves.

• Step 2: Solving the Problem

1) For the first question, we have 12 different books to arrange. The number of possible permutations is 12!

$12!=12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1=479001600$

For the second question, we must arrange the books by subject.

- The 2 math books can be arranged in 2 ways.

- The 4 economics books can be arranged in 4 ways.

- The 6 accounting books can be arranged in 6 ways.

- In addition, we can arrange the 3 groups (math, economics, accounting) in 3 ways.

Therefore, the total number of arrangements is $2!\times4!\times6!\times3!$.

$2!=2\times1=2$

$4!=4\times3\times2\times1=24$

$6!=6\times5\times4\times3\times2\times1=720$

Exercise 04

How many distinct permutations can be formed with all the letters of the words: Time - anabas - sociological - statistics ?

Solution

Step 1: Understanding and analyzing the problem

We need to calculate the number of distinct permutations for each word, taking into account repeated letters. The general formula for the number of permutations of an n-letter word with repetitions is:

$$\frac{n!}{n1!.n2!.n3!\ldots\ldots nk!}$$

Time:

- Number of letters: 4
- Repeated letters: none
- Number of permutations:

4!=4×3×2×1=24

Anabas:

Number of letters: 6

- Repeated letters: a*3 -
- Number of permutations: $\frac{6x5x4x3x2x1}{3x2x1}$ = 120

Sociological:

- Number of letters: 12
- Repeated letters: o (3 times), c (2 times), i (2 times), s (1 time), l (1 time), g (1 time), a (1 time)
- Number of permutations $\frac{12!}{3!2!2}$ = 19958400

Statistics:

- Number of letters: 10
- Repeated letters: s (3 times), t (3 times), a (1 time), i (2 times), c (1 time)
- Number of permutations: $\frac{10!}{3!2!3!}$ = 50400

Exercise 05

1) How many ways can a jury of 3 men and 2 women are formed from 8 mens and 5 womens?

Solution

Step 1: Understanding and Analyzing the Problem

This is a combination problem, as the order in which the jury members are chosen is irrelevant. We must choose 3 men from 8 and 2 women from 5. We will use the following combination formula:

C(n,k) = $\frac{n!}{k!(n-k)!}$

where

- n is the total number of elements and
- k is the number of elements to choose.

• Step 2: Solution

1. Number of ways to choose 3 men out of 8:

$C(8,3)=\frac{8!}{3!(5-3)!} = 56$

2. Number of ways to choose 2 women out of 5:

$C(5,2) = \frac{5!}{2!(5-2)!} = 10$

Exercise 06

An urn contains 6 white balls and 4 black balls. we take 3 balls at once (in the same time) what is the number of ways:

1) To take 3 balls ?
2) To take 3 white balls ?
3) To take 1 white ball and 2 black balls ?

Solution

1. To take 3 balls

Total number of balls:6+4=10

- Number of balls to choose from: 3

- Number of ways $C(10,3) = 120$

2. To take 3 white balls

Number of white balls: 6

- Number of white balls to choose from: 3

- Number of ways: $C(6,3) = 20$

3. To take 1 white ball and 2 black balls

Number of ways to choose 1 white ball from 6:

$C(6,1) = 6$

Number of ways to choose 2 black balls from 4:

$C(4, 2) = 6$

Total number of ways:

$C(6,1)\times C(4,2)=6\times 6=36$

Arrangement, Permutation and Combination

Exercice n° 01 :

In The oral examination in statistics and probability of a DEUG is organized in batches of 3 subjects drawn at random from 80 subjects relating to this course. The student must cover one of the subjects of his choice.

1°/ How many different tests can we organize?

Solution

We are looking for the number of possible combinations of 3 subjects out of 80. The order of the subjects does not matter, so we use the combination formula:

$C(80,3)= \frac{80!}{3!(80-3)!}$

$= 82160$

Exercice n° 02 :

***We** want to form a committee of 7 people, including 2 Republicans, 2 Democrats and 3 independents. We have the choice among 5 Republicans, 6 Democrats and 4 independents. How many ways can we do this?*

Solution

Number of ways to choose 2 Republicans out of 5:

$C(5,2)= 10$

Number of ways to choose 2 Democrats out of 6:

C(6,2)= 15

Number of ways to choose 3 independent options from 4:

C(4,3) =4

To obtain the total number of ways to form the committee, we multiply the number of ways to choose Republicans, Democrats, and Independents:

$C(5,2)\times C(6,2)\times C(4,3)=10\times 15\times 4=600$

Exercice n° 03 :

A student must answer 7 from 10 questions on an exam.

a) *How many ways can he choose them?*

b) Same question if he is forced to choose at least 3 of the first 5 questions?

Solution

We are looking for the number of ways to choose 7 questions from 10 without constraints. This is a combination problem.

b) We must consider the cases where the student chooses 3, 4, or 5 questions from the first 5, and completes their choice with the remaining questions from the last 5.

a) Number of ways to choose 7 questions from 10:

$C(10,7)= 120$

Case 1: 3 questions from the first 5 and 4 from the last 5:

C(5, 3) × C(5, 4) = 50

Case 2: 4 questions from the first 5 and 3 from the last 5:

C(5, 4) ×C(5, 3) = 50

Case 3: 5 questions from the first 5 and 2 from the last 5:

$C(5, 5) \times C(5, 2) = 10$

The number of ways to choose 7 questions with at least 3 among the first 5 is 110. (50+50+10)

<u>Exercice n° 04 :</u>

A class at the school received 4 tickets to the circus. Knowing that this class is made up of 19 students, calculate the number of ways to distribute these 4 tickets in each of the following cases:

a) tickets are numbered and each student can only receive one ticket;

b) the tickets are numbered and each student can receive several tickets;

c) tickets are not numbered and each student can only receive one ticket.

<u>Solution</u>

Understanding and Analyzing the Problem

a) The tickets are numbered, and each student can only receive one ticket. This is an arrangement problem because the order matters (ticket 1 is not the same as ticket 2).

b) The tickets are numbered, and each student can receive multiple tickets. Each ticket can be assigned to any student, regardless of the other tickets.

c) The tickets are not numbered, and each student can only receive one ticket. This is a combination problem because the order does not matter.

a) Numbered tickets, one ticket per student:

This is an arrangement of 4 students out of 19.

$A(19,4)=93024$

Numbered tickets, multiple tickets per student:

Each ticket can be assigned to one of the 19 students. Since there are 4 tickets, the number of possibilities is:

$19\times19\times19\times19=19^4=130321$

c) Unnumbered tickets, one ticket per student:

This is a combination of 4 students out of 19

$C(19,4)= 3876$

<u>Exercice n° 05 :</u>

A mistress of the house has eleven very close friends. She wants to invite five of them to dinner.

a) How many different groups of guests are there?

b) How many possibilities are there if two of them are married and can only come together?

<u>Solution</u>

Understanding and analyzing the problem

a) We are looking for the number of ways to choose 5 friends from 11 without any constraints. This is a combination problem.

b) We must consider two cases: either the two married friends are invited together, or they are not invited at all.

a) Number of ways to choose 5 friends from 11:

$C(11,5) = 462$

Case where both married friends are invited together:

If both married friends are invited, 3 friends still need to be chosen from the remaining 9:

$C(9,3) = 84$

Case where both married friends are not invited:

If both married friends are not invited, you must choose 5 friends from the remaining 9:

$C(9,5)= 126$

We add the results of the two cases:

$84+126=210$

<u>Exercice n° 06 :</u>

In a group there are 10 men, 8 women and 7 children. How many different ways can they be placed on a line if

a) they can position themselves freely?

b) men want to stay together?

Solution

Understanding and analyzing the problem

a) We are looking for the number of possible permutations of all people without constraints.

b) We consider the men as a single group and permute this group with the women and children. Then, we permute the men among themselves.

a) Total number of people:

10+8+7=25

Number of possible permutations:

25! =1.551121004

We consider the 10 men as a single group. Therefore, we have 1 group of men, 8 women, and 7 children, for a total of:

1+8+7= 16 elements to be permuted.

Number of permutations of these 16 elements: 16!

16!=20922789888000

Furthermore, the 10 men can be interchanged within the block:

10!=3628800

The total number of permutations is therefore:

16!x10!

$6!\times10!=20922789888000\times3628800=7.594624276\times10^{16}$

Exercice n° 07 :

A teacher has 10 identical stickers and wants to distribute them among 5 students**.**

Each student can receive any number of stickers*?*

Solution

Understanding and Analyzing the Problem

This is a distribution problem where each toy can be given to any students. We assume the stickers are distinct. Each stickers has 5 options (the 5 students).

Each stickers can be given to one of the 5 students. Since there are 10 stickers, the number of possibilities is:

5x5x5x5x5x5x5x5x5x5=9765625

Calculation of probabilities

Exercise No. 01:

We roll a **coin** and a **die** at the same time. Let the following events be:

A: "Heads and an even number appear"

B: "A number greater than 4 appears on the die"

C: "Tails and an odd number appear"

a. Express the events $A, B, C, \overline{A}, \overline{B}, A - \overline{B}$.

b. Express the events explicitly: A or B, B and C.

c. Which of the events A, B and C are mutually exclusive?

Exercise No. 02:

We roll a die 100 times , the following table gives the six possible results as well as their frequency of appearance:

Number	1	2	3	4	5	6
Frequency	10	19	24	16	15	16

Calculate the probability of the following events:

a. A three appears.

b. C. An even number appears.

c. a five appears.

d. A prime number appears.

Exercise No. 03:

Two well-balanced coins are thrown **.**

Calculate the probability that at least one of the coins resulted in a "tails".

Exercise No. 04:

A school organizes a survey about students' favorite types of transport. In a group of **20 students**, we have:

- 8 students who prefer **car transport**,
- 7 students who prefer **bus transport**,
- 5 students who prefer **bicycle transport**.

We randomly select **3 students simultaneously** from the group.

Calculate the probability of the following events:

a) A = "the three selected students prefer cars"

b) B= "we selected one student for each type of transport"

c) C = "none of the three selected students prefers cars"

d) D = "at least one selected student prefers cars"

e) E = "at least one selected student prefers bus transport"

f) F = "at most one selected student prefers bus transport"

Exercise No. 05:

A cinema accepts two types of membership cards for discounts: **Gold cards** and **Silver cards**.

- 30% of customers have a **Gold card**
- 50% of customers have a **Silver card**
- 15% of customers have **both cards**

A customer is chosen at random.

. What percentage of customers have a credit card accepted by the cinema?

Exercise N 0 6:

A stationery box contains different pens: 4 blue pens, 6 black pens, 10 red pens.

A pen is randomly selected from the box. We define the following events:

- **A**: "the pen is blue" - **B**: "the pen is black" - **C**: "the pen is red"

1. Determine the probabilities p(A) then p(B) and p(C).

2. Represents the experiment using a weighted tree (the associated probability is shown on each branch).

Exercise N 07:

In a university library survey, 600 students were interviewed about their use of digital resources.

- 380 students use **online books**,
- 300 students use **scientific journals**,
- 250 students use **educational videos**,
- 180 students use both **online books and scientific journals**,
- 120 students use both **online books and educational videos**,
- 100 students use both **scientific journals and educational videos**,
- 60 students use **all three resources**.

A student is chosen at random from the survey.

a) What is the probability that the selected student uses **exactly two of the three resources**?

b) What is the probability that the selected student uses **at least one of the three resources**?

Exercise No. 08:

A *travel agency is carrying out a statistical survey on knowledge of three countries A, B, C: Argentina, Brazil and Ca. We see that among the people questioned, 45% know A, 50% know B, 40% know C, 20% know A and B, 15% know A and C, 15% know B and C, 10% know the three countries . A trip is planned for one of the people who responded to the survey.*

The winner is drawn at random. What is the probability that the winner is a person:

a) knowing at least one of these three countries?

b) not knowing any of these three countries?

c) knowing exactly two of the three countries?

d) knowing A, but knowing neither B nor C?

e) knowing A and B but not knowing C?

Exercise No. 09:

A committee of 6 people must be chosen from 20 men and 10 women, what is the probability

a) that it is made up of 6 women?

b) that it consists of 4 men and 2 woman?

b) that it is made up of at least 3 women?

Conditional probability

Exercise No. 01:

A pair of well-balanced dice are thrown. Calculate the probability that the sum obtained is greater than or equal to 10 knowing that:

a. The first die rolled 5.
b. At least one of the dice rolled 5.

Exercise No. 02:

I n a city, 40% of the population has brown hair, 25% has brown eyes and 15% has both characteristics simultaneously. We choose a person at random.

a) What is the probability that this person has neither brown hair nor brown eyes?
b) What is the probability that a person with brown eyes has brown hair?

Exercise No. 03:

A recent graduate plans to take three science exams this coming summer. She will pass the first in June. If she succeeds, she will take the second in July. Then, if she passes this exam, she will take the last one in September. If she does not pass, she is not allowed to take the next exam. The probability of passing the first exam is 0.9. If she continues, the conditional probability of passing the second is 0.8. If she passes the first and second exams, the conditional probability of passing the third is 0.7.

a) With what probability will the candidate pass all three exams?

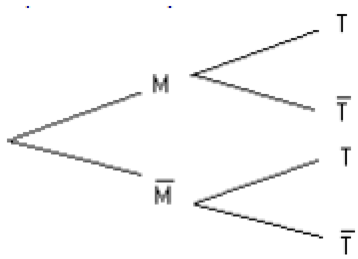
b) Knowing that she will not pass all three exams, what is the probability that she will have failed the second?

Conditional probability; Independence and Bayes' Theorem

Exercise No. 01:

I n a household appliances store, we are interested in the behavior of a potential buyer of a television and a VCR. The probability that he buys a television is 0.6. The probability that he bought a VCR when he bought a television is 0.4. The probability that he buys a VCR when he has not bought a television is 0.2.

1) What is the probability that he will buy a television and a VCR?
2) What is the probability that he will buy a VCR?
3) The customer buys a VCR. How likely is he to buy a television?
4) Complete the following probability tree:



Exercise No. 02:

A car insurance company takes stock of intervention costs among its traffic accident files.

85% of cases result in material repair costs. 20% of cases result in personal injury costs. Among the cases resulting in material repair costs, 12% result in bodily injury costs.

Or the following events:

A: "the processed file incurs material repair costs";

D: "the processed file results in bodily injury costs".

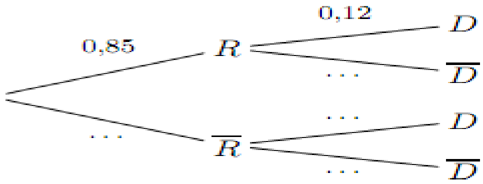
We choose a folder at random.

1. has. Copy and complete the table.

2.

	R	$\overline{R}$	Total
D			
$\overline{D}$			
Total	85		100

b. Copy and complete the weighted tree.



2. We choose a folder at random. Calculate the probability that a file:

a. results in material repair costs and bodily injury costs;

b. only entails material repair costs;

c. only entails bodily injury costs;

d. does not entail any material repair costs or bodily injury costs;

e. entails material repair costs knowing that it entails bodily injury costs.

3. We note that 40% of the cases handled correspond to speeding and of these, 60% result in bodily injury costs.

We note E: "the processed file corresponds to excessive speed".

a. We choose a file. What is the probability that this case corresponds to excessive speed and results in bodily injury costs?

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Independence in probabilities

Exercise No. 01:

We are interested in a family and we consider the events:
'A': the family has children of both sexes,
'B': the family has at most one boy.

a) Show that A and B are dependent if the family has two children.
b) Show that A and B are independent if the family has three children.

Exercise No. 02:

We have 10 cards numbered 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
1.We draw two cards and Each time **we put the card back into the deck.** Calculate the probability

a) get 5 and 6 in that order;
b) to obtain 5 and 6 if the order does not play a role
c) to obtain 6 exactly once;
d) to obtain 6 at least once.

3. If two cards are drawn **without putting them back** in the pack. Calculate the probability P(A); P(B); P(C) and P(D).

Exercice N03

The following table gives the distribution of 150 trainees according to the language chosen and the sporting activity chosen.

	Tennis	Horse riding	sailing
English	45	18	27
German	33	9	18

1) We choose a student at random.
2) Are the "study German" and "play tennis" events independent?
3) Are the "study English" and "practice sailing" events independent?

Conclusion:

The *Probability 1* module is intended for first-year preparatory class students and aims to provide a solid introduction to the fundamental principles of probability theory. This module helps students develop logical reasoning and analytical skills necessary for understanding and solving problems involving uncertainty and random phenomena.

Throughout this course, students are introduced to key concepts such as sample spaces, events, probability laws, conditional probability, independence, and random variables. They also learn how to apply mathematical methods to calculate probabilities and interpret results in different practical situations.

By mastering these concepts, students build a strong mathematical foundation that will support their future studies in advanced probability, economics, and other scientific disciplines. Moreover, this module enhances problem-solving abilities and encourages a rigorous scientific approach, which is essential for academic success in preparatory studies and beyond.

The reviewer

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